Can the lepton flavor mixing matrix be symmetric?

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Current neutrino oscillation data indicate that the 3×3 lepton flavor mixing matrix V is likely to be symmetric about its V_{e3}-$V_{\mu2}$-$V_{\tau1}$ axis. This off-diagonal symmetry corresponds to three pairs of *congruent* unitarity triangles in the complex plane. Terrestrial matter effects can substantially modify the genuine CP-violating parameter and off-diagonal asymmetries of V in realistic long-baseline experiments of neutrino oscillations.

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I. INTRODUCTION

The observed anomalies of atmospheric [1] and solar [2] neutrinos strongly suggest that neutrinos are massive and lepton flavors are mixed. In the framework of three charged leptons and three active neutrinos, the phenomena of flavor mixing and CP violation are described by a unitary matrix V, which relates the neutrino mass eigenstates (ν_1,ν_2,ν_3) to the neutrino flavor eigenstates (ν_e,ν_μ,ν_τ):

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\begin{pmatrix}V_{e1} & V_{e2} & V_{e3}\\ V_{\mu1} & V_{\mu2} & V_{\mu3}\\ V_{\tau1} & V_{\tau2} & V_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}. \tag{1}$$

The unitarity of V represents two sets of normalization and orthogonality conditions:

$$\sum_i (V_{\alpha i}V^*_{\beta i})=\delta_{\alpha\beta},$$

$$\sum_\alpha (V_{\alpha i}V^*_{\alpha j})=\delta_{ij}, \tag{2}$$

where Greek and Latin subscripts run over (e,μ,τ) and $(1,2,3)$, respectively. If neutrinos are Dirac particles, a full parametrization of V requires four independent parameters—three mixing angles and one CP-violating phase, for example. If neutrinos are Majorana particles, however, two additional CP-violating phases need to be introduced for a complete parametrization of V. In both cases, CP and T violation in normal neutrino oscillations depends only upon a single rephasing-invariant parameter $\mathcal{J}$ [3], defined through

$$\text{Im}(V_{\alpha i}V_{\beta j}V^*_{\alpha j}V^*_{\beta i})=\mathcal{J}\sum_{\gamma,k}(\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}), \tag{3}$$

where (α,β,γ) and (i,j,k) run, respectively, over (e,μ,τ) and $(1,2,3)$. A major goal of the future long-baseline neutrino oscillation experiments is to measure $|V_{\alpha i}|$ and $\mathcal{J}$ as precisely as possible [4]. Once the matrix elements of V are determined to a good degree of accuracy, a stringent test of its unitarity will become available.

As a straightforward consequence of the unitarity of V, two interesting relations can be derived from the normalization conditions in Eq. (2):

$$|V_{e2}|^2-|V_{\mu1}|^2=|V_{\mu3}|^2-|V_{\tau2}|^2=|V_{\tau1}|^2-|V_{e3}|^2\equiv\Delta_L \tag{4}$$

and

$$|V_{e2}|^2-|V_{\mu3}|^2=|V_{\mu1}|^2-|V_{\tau2}|^2=|V_{\tau3}|^2-|V_{e1}|^2\equiv\Delta_R. \tag{5}$$

The off-diagonal asymmetries Δ_L and Δ_R characterize the geometrical structure of V about its V_{e1}-$V_{\mu2}$-$V_{\tau3}$ and V_{e3}-$V_{\mu2}$-$V_{\tau1}$ axes, respectively. If $\Delta_L=0$ held, V would be symmetric about the V_{e1}-$V_{\mu2}$-$V_{\tau3}$ axis. Indeed, the counterpart of Δ_L in the quark sector is very small (of order 10^{-5} [5]), i.e., the 3×3 quark mixing matrix is almost symmetric about its V_{ud}-V_{cs}-V_{tb} axis. An exactly symmetric flavor mixing matrix may hint at an underlying flavor symmetry, from which some deeper understanding of the fermion mass texture can be achieved [6]. In this sense, the tiny off-diagonal asymmetry of the quark flavor mixing matrix is likely to arise from a slight breakdown of certain flavor symmetries of quark mass matrices.

The purpose of this paper is to examine whether or not the lepton flavor mixing matrix V is really symmetric. In Sec. II, we find that current neutrino oscillation data strongly favor $\Delta_R=0$, i.e., V may be symmetric about its V_{e3}-$V_{\mu2}$-$V_{\tau1}$ axis. It is too early to get any phenomenological constraints on Δ_L, unless very special assumptions are made. In Sec. III, we point out that the off-diagonal symmetry $\Delta_R=0$ corresponds to three pairs of *congruent* unitarity triangles in the complex plane. Taking realistic long-baseline experiments of neutrino oscillations into account, the terrestrial matter effects on $\mathcal{J}$, Δ_L, and Δ_R are briefly discussed in Sec. IV. Section V is devoted to some further discussions about possible implications of $\Delta_R=0$ on specific textures of lepton mass matrices. Finally, we summarize our main results in Sec. VI.

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II. OFF-DIAGONAL SYMMETRY

Current experimental data [1,2] strongly favor the hypothesis that atmospheric and solar neutrino oscillations are dominated by $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\mu$ transitions, respectively. Thus their mixing factors $\sin^2 2\theta_{\rm atm}$ and $\sin^2 2\theta_{\rm sun}$ have rather simple relations with the elements of the lepton flavor mixing matrix V. The mixing factor associated with the CHOOZ (or Palo Verde) reactor neutrino oscillation experiment [7], denoted as $\sin^2 2\theta_{\rm chz}$, is also a simple function of $|V_{\alpha i}|$ in the same hypothesis. The explicit expressions of $\sin^2 2\theta_{\rm sun}$, $\sin^2 2\theta_{\rm atm}$, and $\sin^2 2\theta_{\rm chz}$ read as follows:

$$\sin^2 2\theta_{\rm sun} = 4|V_{e1}|^2|V_{e2}|^2,$$

$$\sin^2 2\theta_{\rm atm} = 4|V_{\mu 3}|^2(1-|V_{\mu 3}|^2), \tag{6}$$

$$\sin^2 2\theta_{\rm chz} = 4|V_{e3}|^2(1-|V_{e3}|^2).$$

An analysis of the Super-Kamiokande data on atmospheric neutrino oscillations [1] yields $0.88 \leqslant \sin^2 2\theta_{\rm atm} \leqslant 1.0$ and 1.6×10^{-3} eV$^2 \leqslant \Delta m^2_{\rm atm} \leqslant 4.0\times 10^{-3}$ eV^2 at the 90% confidence level. Corresponding to $\Delta m^2_{\rm chz} > 2.0\times 10^{-3}$ eV^2, $\sin^2 2\theta_{\rm chz} < 0.18$ can be drawn from the CHOOZ experiment [7]. We restrict ourselves to the large-angle Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar neutrino problem [8], as it gives the best global fit of the present data. At the 99% confidence level, $0.56 \leqslant \sin^2 2\theta_{\rm sun} \leqslant 0.99$ and 2.0×10^{-5} eV$^2 \leqslant \Delta m^2_{\rm sun} \leqslant 5.0\times 10^{-4}$ eV^2 have been obtained [9].

With the help of Eqs. (2) and (6), one may express $|V_{e1}|^2$, $|V_{e2}|^2$, $|V_{e3}|^2$, and $|V_{\mu 3}|^2$ in terms of $\theta_{\rm sun}$, $\theta_{\rm atm}$, and $\theta_{\rm chz}$:

$$|V_{e1}|^2 = \frac{1}{2}\left(\cos^2\theta_{\rm chz} \pm \sqrt{\cos^4\theta_{\rm chz} - \sin^2 2\theta_{\rm sun}}\right),$$

$$|V_{e2}|^2 = \frac{1}{2}\left(\cos^2\theta_{\rm chz} \mp \sqrt{\cos^4\theta_{\rm chz} - \sin^2 2\theta_{\rm sun}}\right), \tag{7}$$

$$|V_{e3}|^2 = \sin^2\theta_{\rm chz} \ \ {\rm or} \ \ \cos^2\theta_{\rm chz},$$

$$|V_{\mu 3}|^2 = \sin^2\theta_{\rm atm} \ \ {\rm or} \ \ \cos^2\theta_{\rm atm}.$$

Without loss of generality, three mixing angles ($\theta_{\rm sun}$, $\theta_{\rm atm}$, and $\theta_{\rm chz}$) can all be arranged to lie in the first quadrant. Then we need only adopt the solution $|V_{e3}|^2 = \sin^2\theta_{\rm chz}$ [10], in accord with $\sin^2 2\theta_{\rm chz} < 0.18$. We may also express $|V_{\tau 3}|^2$ in terms of $\theta_{\rm atm}$ and $\theta_{\rm chz}$, once the normalization relation $|V_{e3}|^2 + |V_{\mu 3}|^2 + |V_{\tau 3}|^2 = 1$ is taken into account. It turns out that useful experimental constraints are achievable for those matrix elements in the first row and in the third column of V.

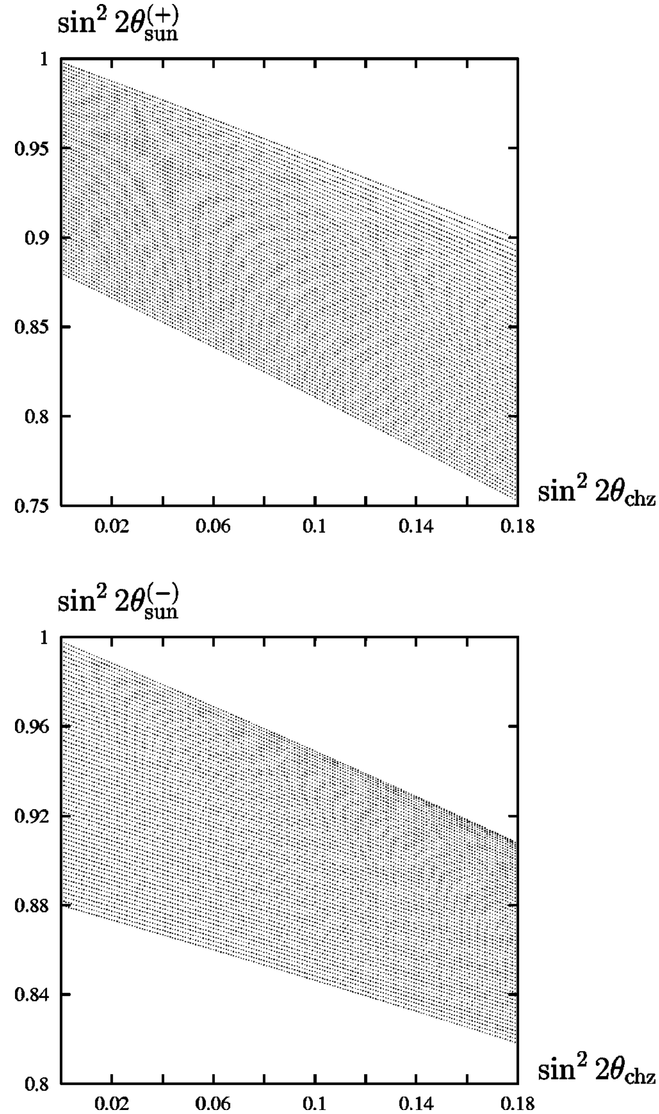

FIG. 1. Dependence of $\sin^2 2\theta^{(\pm)}_{\rm sun}$ on $\sin^2 2\theta_{\rm chz}$, where $0.88 \leqslant \sin^2 2\theta_{\rm atm} \leqslant 1.0$ has been input.

However, it is impossible to get any constraints on the other four matrix elements of V, unless some special assumptions are made.[1] This observation means that it remains too early to get any instructive information on the off-diagonal asymmetry Δ_L from current neutrino oscillation experiments, but it is already possible to examine whether $\Delta_R = 0$ coincides with current data and what its implications can be on leptonic CP violation and unitarity triangles.

To see whether $\Delta_R = 0$ is compatible with the present data of solar, atmospheric, and reactor neutrino oscillations, we simply set $|V_{e2}|^2 = |V_{\mu 3}|^2$ in Eq. (7) and then obtain

[1]Allowing the Dirac-type CP-violating phase of V to vary between 0 and π, Fukugita and Tanimoto [11] have presented the numerical ranges of all nine $|V_{\alpha i}|$ by use of current neutrino oscillation data. This rough construction of the lepton flavor mixing matrix is actually unable to shed light on its off-diagonal asymmetries and CP-violating features.

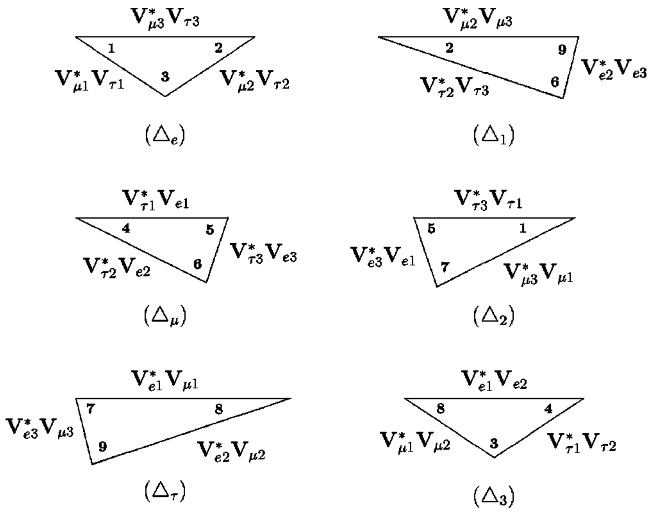

FIG. 2. Unitarity triangles of the lepton flavor mixing matrix in the complex plane. Each triangle is named by the Greek or Latin subscript that does not manifest in its three sides.

$$\sin^2 2\theta^{(\pm)}_{\rm sun} = \sin^2 2\theta_{\rm atm} - (1 \pm \sqrt{1-\sin^2 2\theta_{\rm atm}}) \times (1-\sqrt{1-\sin^2 2\theta_{\rm chz}}). \tag{8}$$

As $\sin^2 2\theta_{\rm chz} < 0.18$ [7], the second term on the right-hand side of Eq. (8) serves as a small correction to the leading term $\sin^2 2\theta_{\rm atm}$. The difference between $\sin^2 2\theta^{(+)}_{\rm sun}$ and $\sin^2 2\theta^{(-)}_{\rm sun}$ is therefore insignificant. Indeed $\sin^2 2\theta_{\rm chz} = 0$ leads definitely to $\sin^2 2\theta^{(+)}_{\rm sun} = \sin^2 2\theta^{(-)}_{\rm sun} = \sin^2 2\theta_{\rm atm}$, as a straightforward result of $\Delta_R = 0$. Allowing $\sin^2 2\theta_{\rm atm}$ to vary in the experimental range $0.88 \leqslant \sin^2 2\theta_{\rm atm} \leqslant 1.0$, we plot the numerical dependence of $\sin^2 2\theta^{(\pm)}_{\rm sun}$ on $\sin^2 2\theta_{\rm chz}$ in Fig. 1. One can observe that the values of $\sin^2 2\theta^{(\pm)}_{\rm sun}$ predicted from Eq. (8) are very consistent with current experimental data. Thus we conclude that a vanishing or tiny off-diagonal asymmetry of V about its V_{e3}-$V_{\mu 2}$-$V_{\tau 1}$ axis is strongly favored.

III. UNITARITY TRIANGLES

Let us proceed to discuss possible implications of $\Delta_R = 0$ on the leptonic unitarity triangles. It is known that six orthogonality relations of V in Eq. (2) correspond to six triangles in the complex plane [6], as illustrated in Fig. 2. These six triangles have a total of 18 different sides and nine different inner angles. Unitarity requires that all six triangles have the same area amounting to $\mathcal{J}/2$, where $\mathcal{J}$ is just the rephasing-invariant measure of leptonic CP violation defined in Eq. (3).

Now that the off-diagonal asymmetries Δ_L and Δ_R describe the geometrical structure of V, they must have direct relations with the unitarity triangles in the complex plane. Indeed, it is easy to show that $\Delta_L = 0$ or $\Delta_R = 0$ corresponds to the congruence between two unitarity triangles, i.e.,

$$\Delta_L = 0 \Rightarrow \triangle_e \cong \triangle_1, \quad \triangle_\mu \cong \triangle_2, \quad \triangle_\tau \cong \triangle_3 \tag{9}$$

and

$$\Delta_R = 0 \Rightarrow \triangle_e \cong \triangle_3, \quad \triangle_\mu \cong \triangle_2, \quad \triangle_\tau \cong \triangle_1. \tag{10}$$

As $\Delta_R = 0$ is expected to be rather close to reality, we draw the conclusion that the unitarity triangles $\triangle_e$ and $\triangle_3$ must be approximately congruent with each other. A similar conclusion can be drawn for the unitarity triangles $\triangle_\mu$ and $\triangle_2$ as well as $\triangle_\tau$ and $\triangle_1$. The long-baseline experiments of neutrino oscillations in the near future will tell whether an approximate congruence exists between $\triangle_e$ and $\triangle_1$ or between $\triangle_\tau$ and $\triangle_3$. A particularly interesting possibility would be $\Delta_L \approx \Delta_R \approx 0$, i.e., only two of the six unitarity triangles are essentially distinct.

Next we examine how large the area of each unitarity triangle (i.e., $\mathcal{J}/2$) can maximally be in the limit $\Delta_R = 0$, in which V is parametrized as follows:

$$V = \begin{pmatrix} c_x c_z & s_x c_z & s_z \\ -c_x s_x s_z - c_x s_x e^{-i\delta} & -s_x^2 s_z + c_x^2 e^{-i\delta} & s_x c_z \\ -c_x^2 s_z + s_x^2 e^{-i\delta} & -c_x s_x s_z - c_x s_x e^{-i\delta} & c_x c_z \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{11}$$

where $s_x \equiv \sin\theta_x$, $c_z \equiv \cos\theta_z$, and so on. The merit of this phase choice is that the Dirac-type CP-violating phase δ does not appear in the effective-mass term of the neutrinoless double beta decay [12], which depends only upon the Majorana phases ρ and σ. Without loss of generality, one may arrange the mixing angles θ_x and θ_z to lie in the first quadrant. Three CP-violating phases (δ, ρ, σ) can take arbitrary values from 0 to 2π. Clearly $\mathcal{J} = c_x^2 s_x^2 c_z^2 s_z \sin\delta$ holds. With the help of Eq. (6) or Eq. (7), we are able to figure out the relations between (θ_x, θ_z) and $(\theta_{\rm sun}, \theta_{\rm chz})$. The result is

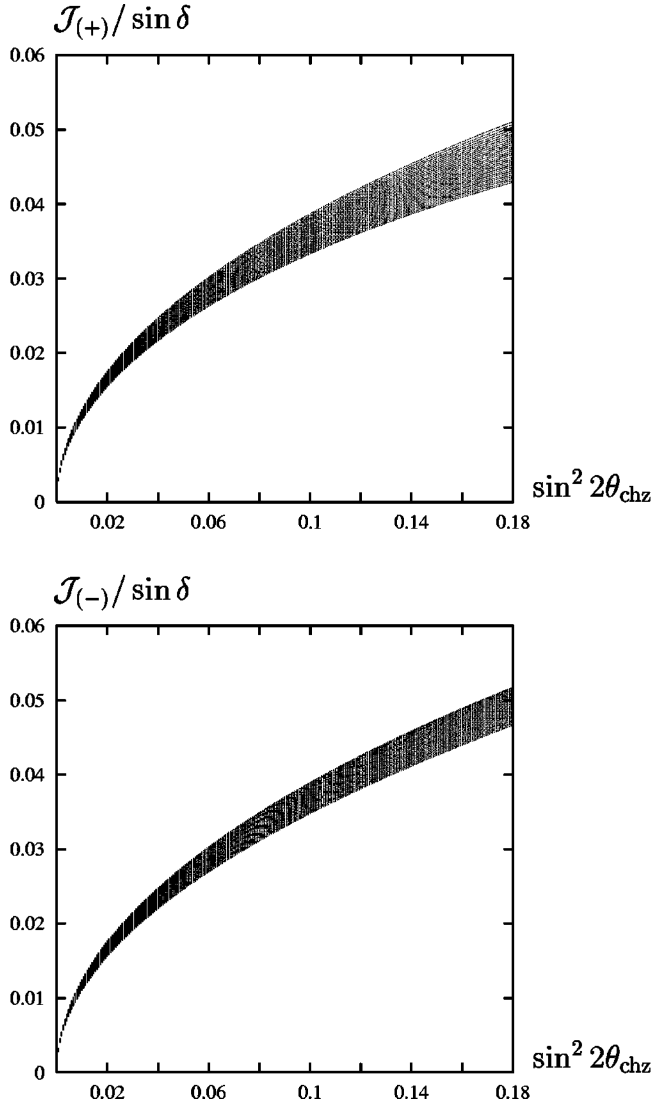

FIG. 3. Dependence of $\mathcal{J}_{(\pm)}/\sin\delta$ on $\sin^2 2\theta_{\rm chz}$, where $0.88 \leqslant \sin^2 2\theta_{\rm atm} \leqslant 1.0$ has been input.

$$\sin^2 2\theta_x = \frac{4\sin^2 2\theta_{\rm sun}}{(1+\sqrt{1-\sin^2 2\theta_{\rm chz}})^2},$$

$$\sin^2 2\theta_z = \sin^2 2\theta_{\rm chz}. \tag{12}$$

Then we obtain

$$\mathcal{J}_{(\pm)} = \frac{\sqrt{2}}{4}\sin^2 2\theta^{(\pm)}_{\rm sun}\sin\delta\frac{\sqrt{1-\sqrt{1-\sin^2 2\theta_{\rm chz}}}}{1+\sqrt{1-\sin^2 2\theta_{\rm chz}}}, \tag{13}$$

where $\sin^2 2\theta^{(\pm)}_{\rm sun}$ has been given in Eq. (8). Again the difference between $\mathcal{J}_{(+)}$ and $\mathcal{J}_{(-)}$ is insignificant. If $\delta=0$ or π held, we would arrive at $\mathcal{J}_{(\pm)}=0$. In general, however, CP symmetry is expected to break down in the lepton sector. For illustration, we plot the numerical dependence of $\mathcal{J}_{(\pm)}/\sin\delta$ on $\sin^2 2\theta_{\rm chz}$ in Fig. 3, where the experimentally allowed values of $\sin^2 2\theta_{\rm atm}$ are used. It is obvious that the upper bound of $\mathcal{J}_{(\pm)}$ (when $\delta=\pi/2$) can be as large as a few percent, only if $\sin^2 2\theta_{\rm chz} \geqslant 0.01$. This result implies that leptonic CP and T violation might be observable in the future long-baseline neutrino oscillation experiments.

IV. MATTER EFFECTS

In realistic long-baseline experiments of neutrino oscillations, the terrestrial matter effects must be taken into account [13]. The pattern of neutrino oscillations in matter can be expressed in the same form as that in vacuum, however, if we define the *effective* neutrino masses $\tilde{m}_i$ and the *effective* lepton flavor mixing matrix $\tilde{V}$ in which the terrestrial matter effects are already included [14]. Note that $\tilde{m}_i$ are functions of m_j, $|V_{ej}|$, and A; and $\tilde{V}_{\alpha i}$ are functions of m_j, $V_{\beta j}$, and A, where A is the matter parameter and its magnitude depends upon the neutrino beam energy E and the background density of electrons N_e. The analytically exact relations between $(\tilde{m}_i, \tilde{V}_{\alpha i})$ and $(m_i, V_{\alpha i})$ can be found in Ref. [14] if N_e is assumed to be a constant. In analogy to the definitions of $\mathcal{J}$, Δ_L, and Δ_R, the *effective* CP-violating parameter $\tilde{\mathcal{J}}$ and off-diagonal asymmetries $\tilde{\Delta}_L$ and $\tilde{\Delta}_R$ can be defined as follows:

$$\text{Im}(\tilde{V}_{\alpha i}\tilde{V}_{\beta j}\tilde{V}^*_{\alpha j}\tilde{V}^*_{\beta i}) = \tilde{\mathcal{J}}\sum_{\gamma,k}(\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}), \tag{14}$$

where (α,β,γ) and (i,j,k) run, respectively, over (e,μ,τ) and (1,2,3),

$$|\tilde{V}_{e2}|^2 - |\tilde{V}_{\mu 1}|^2 = |\tilde{V}_{\mu 3}|^2 - |\tilde{V}_{\tau 2}|^2 = |\tilde{V}_{\tau 1}|^2 - |\tilde{V}_{e3}|^2 \equiv \tilde{\Delta}_L \tag{15}$$

and

$$|\tilde{V}_{e2}|^2 - |\tilde{V}_{\mu 3}|^2 = |\tilde{V}_{\mu 1}|^2 - |\tilde{V}_{\tau 2}|^2 = |\tilde{V}_{\tau 3}|^2 - |\tilde{V}_{e1}|^2 \equiv \tilde{\Delta}_R. \tag{16}$$

It is then interesting to examine possible departures of $(\tilde{\mathcal{J}}, \tilde{\Delta}_L, \tilde{\Delta}_R)$ from $(\mathcal{J}, \Delta_L, \Delta_R)$ in a concrete experimental scenario.

To illustrate, we compute $\tilde{\mathcal{J}}$, $\tilde{\Delta}_L$ and $\tilde{\Delta}_R$ using the typical inputs $\theta_x=40°$, $\theta_z=5°$, and $\delta=90°$, which yield $\mathcal{J}=0.021$, $\Delta_L=0.166$, and $\Delta_R=0$ in vacuum. The neutrino mass-squared differences are taken to be $\Delta m^2_{21}=\Delta m^2_{\rm sun}=5\times10^{-5}$ eV^2 and $\Delta m^2_{31}=\Delta m^2_{\rm atm}=3\times10^{-3}$ eV^2. The explicit formulas relevant to our calculation have been given in Ref. [14]. We plot the numerical dependence of $\tilde{\mathcal{J}}/\mathcal{J}$, $\tilde{\Delta}_L$, and $\tilde{\Delta}_R$ on the matter parameter A in Fig. 4, where both the cases of neutrinos $(+A)$ and antineutrinos $(-A)$ are taken into account. One can see that the off-diagonal symmetry $\Delta_R=0$ in vacuum can substantially be spoiled by terrestrial matter effects. The deviation of $\tilde{\Delta}_L$ from Δ_L and that of $\tilde{\mathcal{J}}$ from $\mathcal{J}$ are remarkably large, if $A>10^{-5}$ eV^2.

As emphasized in Ref. [15], there exists the simple reversibility between the fundamental neutrino mixing parameters in vacuum and their effective counterparts in matter. The former can therefore be expressed in terms of the latter,

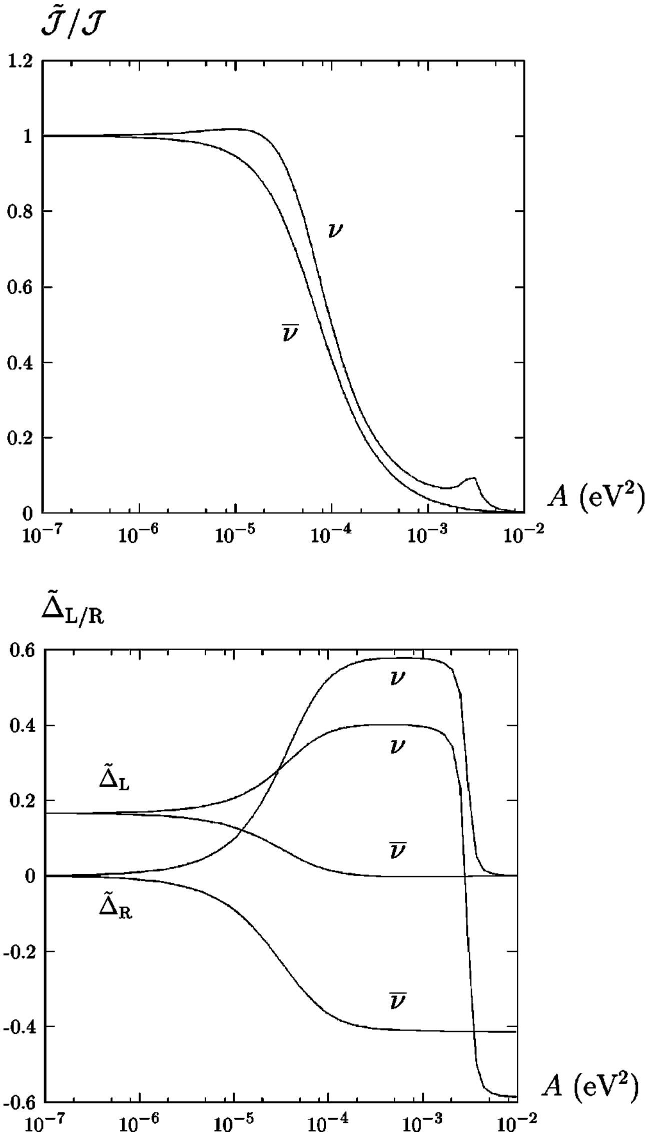

FIG. 4. Illustrative plots for terrestrial matter effects on $\mathcal{J}$, Δ_L, and Δ_R associated with neutrinos $(+A)$ and antineutrinos $(-A)$, where $\Delta m^2_{21}=5\times10^{-5}$ eV^2, $\Delta m^2_{31}=3\times10^{-3}$ eV^2, $\theta_x=40°$, $\theta_z=5°$, and $\delta=90°$ have typically been input.

allowing more straightforward extraction of the genuine lepton mixing quantities (including $\mathcal{J}$, Δ_L, and Δ_R) from a variety of long-baseline neutrino oscillation experiments. If $|V_{\alpha i}|$ are determined to a very high degree of accuracy, it will be possible to test the unitarity of V [6] and establish leptonic CP violation through the nonzero area of six unitarity triangles [16] even in the absence of a direct measurement of $\mathcal{J}$.

V. FURTHER DISCUSSIONS

If $\Delta_R=0$ really holds, one may wonder whether this off-diagonal symmetry of V hints at very special textures of the neutrino mass matrix M_ν and (or) the charged lepton mass matrix M_l. In the following, we take two simple but instructive examples to illustrate possible implications of $\Delta_R=0$ on M_l and M_ν.

Example A

In no conflict with current data on atmospheric [1], solar [2], and reactor [7] neutrino oscillations, a remarkably simplified form of V with $\Delta_R=0$ and $\delta=\rho=\sigma=0$ is

$$\mathbf{V}=\begin{pmatrix} \frac{c}{\sqrt{2}} & \frac{c}{\sqrt{2}} & s \\ -\frac{1+s}{2} & \frac{1-s}{2} & \frac{c}{\sqrt{2}} \\ \frac{1-s}{2} & -\frac{1+s}{2} & \frac{c}{\sqrt{2}} \end{pmatrix}, \tag{17}$$

where $s\equiv\sin\theta\ll1$ and $c\equiv\cos\theta\approx1$ [17]. In the flavor basis where M_l is diagonal, M_ν can be given as

$$M_\nu=\mathbf{V}\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}\mathbf{V}^{\rm T}=m_1\mathbf{N}_1+m_2\mathbf{N}_2+m_3\mathbf{N}_3, \tag{18}$$

where symmetric matrices $\mathbf{N}_1$, $\mathbf{N}_2$, and $\mathbf{N}_3$ read

$$\mathbf{N}_1=\begin{pmatrix} \frac{c^2}{2} & -\frac{c(1+s)}{2\sqrt{2}} & \frac{c(1-s)}{2\sqrt{2}} \\ & \frac{(1+s)^2}{4} & -\frac{1-s^2}{4} \\ & & \frac{(1-s)^2}{4} \end{pmatrix},$$

$$\mathbf{N}_2=\begin{pmatrix} \frac{c^2}{2} & \frac{c(1-s)}{2\sqrt{2}} & -\frac{c(1+s)}{2\sqrt{2}} \\ & \frac{(1-s)^2}{4} & -\frac{1-s^2}{4} \\ & & \frac{(1+s)^2}{4} \end{pmatrix}, \tag{19}$$

$$\mathbf{N}_3=\begin{pmatrix} s^2 & \frac{cs}{\sqrt{2}} & \frac{cs}{\sqrt{2}} \\ & \frac{c^2}{2} & \frac{c^2}{2} \\ & & \frac{c^2}{2} \end{pmatrix}.$$

The texture of M_ν is rather complicated, hence it is difficult to observe any hidden flavor symmetry associated with lepton mass matrices.

Example B

The result of M_ν in example A will become simpler if $s=0$ is taken further (i.e., $\mathbf{V}$ is of the bimaximal mixing form

[18]). In this special case, however, simple textures of M_l and M_ν can be written out in a more general flavor basis. It is easy to show that M_l and M_ν of the following textures lead to **V** with $s=0$:

$$M_l = \frac{C_l}{2}\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \begin{pmatrix} \delta_l & 0 & 0 \\ 0 & 0 & \varepsilon_l \\ 0 & \varepsilon_l & 0 \end{pmatrix}\right],$$

$$M_\nu = \frac{C_\nu}{2}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \varepsilon_\nu & 0 \\ \varepsilon_\nu & 0 & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}\right], \tag{20}$$

where $\delta_{l,\nu}$ and $\varepsilon_{l,\nu}$ are small perturbative parameters [19]. In the limit $\delta_{l,\nu}=\varepsilon_{l,\nu}=0$, M_l has the $S(2)_L \times S(2)_R$ symmetry and M_ν displays the $S(3)$ symmetry [20]. The perturbative corrections in M_l allow electrons and muons to acquire their masses:

$$\{m_e, m_\mu, m_\tau\} = \frac{C_l}{2}\{|\delta_l|, |\varepsilon_l|, 2+\varepsilon_l\}. \tag{21}$$

We then arrive at $C_l = m_\mu + m_\tau \approx 1.88$ GeV, $|\varepsilon_l| = 2m_\mu/(m_\mu+m_\tau) \approx 0.11$, and $|\delta_l| = 2m_e/(m_\mu+m_\tau) \approx 5.4 \times 10^{-4}$. The perturbative corrections in M_ν make three neutrino masses nondegenerate:

$$\{m_1, m_2, m_3\} = C_\nu\{1+\varepsilon_\nu, 1-\varepsilon_\nu, 1+\delta_\nu\}. \tag{22}$$

As a result, $|\varepsilon_\nu|/|\delta_\nu| \approx \Delta m^2_{\rm sun}/(2\Delta m^2_{\rm atm}) \sim 10^{-2}$ for the large-angle MSW solution to the solar neutrino problem.

Examples A and B illustrate how the off-diagonal symmetry of V $(\Delta_R=0)$ can be reproduced from specific textures of lepton mass matrices.

VI. SUMMARY

In view of current experimental data on solar and atmospheric neutrino oscillations, we have discussed the geometrical structure of the 3×3 lepton flavor mixing matrix V. We find that the present data strongly favor the off-diagonal symmetry of V about its V_{e3}-$V_{\mu 2}$-$V_{\tau 1}$ axis. This symmetry, if it really exists, will correspond to three pairs of congruent unitarity triangles in the complex plane. It remains too early to tell or not whether V is symmetric about its V_{e1}-$V_{\mu 2}$-$V_{\tau 3}$ axis. A brief analysis of terrestrial matter effects on the universal CP-violating parameter and off-diagonal asymmetries of V has also been made. We expect that future long-baseline experiments of neutrino oscillations can help establish the texture of the lepton flavor mixing matrix, from which one could get some insights into the underlying flavor symmetries responsible for the charged lepton and neutrino mass matrices.

ACKNOWLEDGMENTS

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